UNIVERSITY OF TEXAS AT AUSTIN
Dept. of Electrical and Computer Engineering

Quiz #1

Date: February 28, 2001 Course: EE 313

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Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework and solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- Fully justify your answers unless instructed otherwise.

Problem	Point Value	Your Score	Topic
1	25		Differential Equation
2	20		Convolution
3	20		Tapped Delay Line
4	20		Step Response
5	15		Discrete-Time Stability
Total	100		

K-12

Problem 1.1 Differential Equation. 25 points.
Given the following differential equation

$$\frac{d^2}{dt^2}y(t) - 2\frac{d}{dt}y(t) + y(t) = f(t)$$

(a) What are the characteristic roots? 5 points.

$\lambda^2 - 2\lambda + 1 = 0$

$(\lambda - 1)^2 = 0 \Rightarrow \lambda = \{1, 1\}$

(b) Find the zero-input response assuming non-zero initial conditions for $y'(0)$ and $y''(0)$. You may leave your answer in terms of C_1 and C_2. 10 points.

$y(t) = C_1 e^t + C_2 t e^t$

$y'(t) = C_1 e^t + C_2 (t e^t + e^t)$

$y'(0) = C_1 + C_2$

$y''(t) = C_1 e^t + C_2 (t e^t + 2 e^t)$

$y''(0) = C_1 + 2C_2$

(c). Find the zero-input response for the initial conditions $y'(0) = 1$ and $y''(0) = -2$. 10 points.

$y'(0) = C_1 + C_2 = 1$

$y''(0) = C_1 + 2C_2 = -2$

$\Rightarrow$ $C_1 = 4$, $C_2 = -3$

$y(t) = 4e^t - 3te^t$

K-13

Problem 1.2 Convolution. 20 points.

Sketch the following convolutions. On the sketches, clearly label significant points on the horizontal and vertical axes. You do not have to show intermediate work, but showing intermediate work may qualify for partial credit.

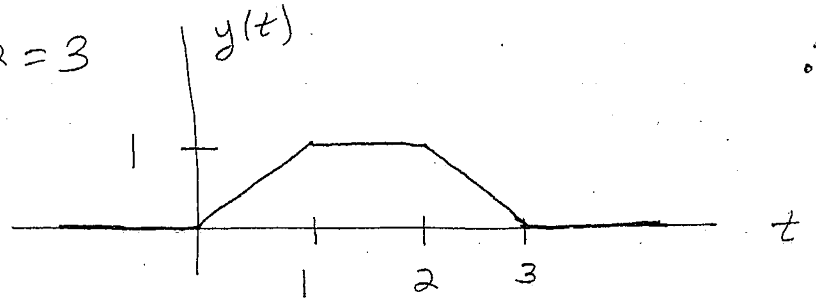
(a) Continuous-time convolution. 10 points.
Sketch $y(t) = p_1(t) * p_2(t)$, where

extent = 1
$$p_1(t) = \begin{cases} 1 & \text{for } 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$
causal.

and

extent = 2
$$p_2(t) = \begin{cases} 1 & \text{for } 0 \le t \le 2 \\ 0 & \text{otherwise} \end{cases}$$
causal

$\therefore$ extent of $y(t) = 1 + 2 = 3$



$\therefore$ y(t) should be causal

(b) Discrete-time convolution. 10 points.
Sketch $y[k] = h[k] * u[k]$, where $h[k]$ is the impulse response of a first-order differencer given by

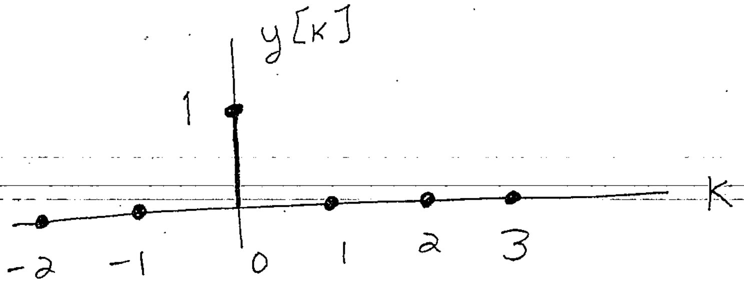
$$h[k] = \begin{cases} 1 & \text{for } k = 0 \\ -1 & \text{for } k = 1 \\ 0 & \text{otherwise} \end{cases}$$

discrete-time version of the differentiator.

$$y[k] = \sum_{n=-\infty}^{\infty} h[k]\, u[k-n]$$

$$y[k] = u[k] - u[k-1]$$

applying a differentiator to the unit step function (in continuous time) gives an impulse.



K-14

Problem 1.3 Tapped Delay Line. 20 points.

A discrete-time tapped delay line is a linear time-invariant system. For input $x[k]$, the output $y[k]$ is

$$y[k] = \sum_{n=0}^{N-1} a_n\, x[k-n]$$

(a) Compute the impulse response $h[k]$. 5 points.

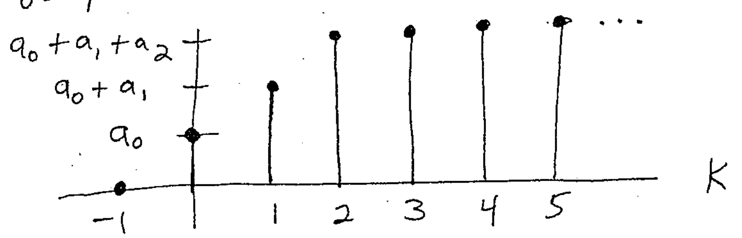
$$h[k] = \sum_{n=0}^{N-1} a_n\, \delta[k-n]$$

(b) Compute the step response, i.e. the response when unit step $u[k]$ is input. 5 points.

$$y_{step}[k] = \sum_{n=0}^{N-1} a_n\, u[k-n]$$

(c) Sketch the step response for $N = 3$. The sketch should be in terms of a_0, a_1, and a_2. 5 points.

$$y_{step}[k] = a_0\, u[k] + a_1\, u[k-1] + a_2\, u[k-2]$$



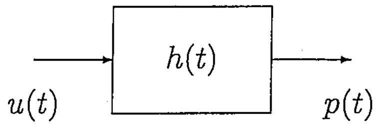
(d) What is the system time constant as a function of N, a_0, a_1, $\ldots$, a_{N-1}? You can answer this based on either the impulse response or the step response. 5 points.

$N-1$ samples

K-15

Problem 1.4 Step Response of Linear Time-Invariant Systems

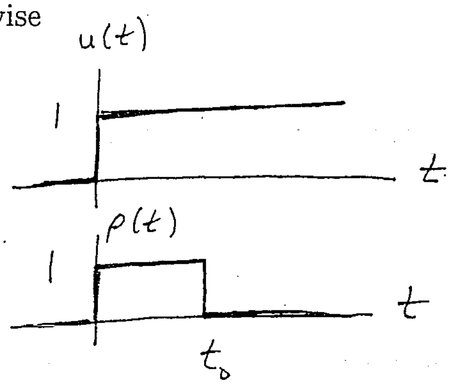
(a) Continuous-Time. 10 points.
For the block diagram below,



$p(t)$ is a pulse that starts at the origin and ends at $t = t_0$:

$$p(t) = \begin{cases} 1 & \text{for } 0 \le t < t_0 \\ 0 & \text{otherwise} \end{cases}$$

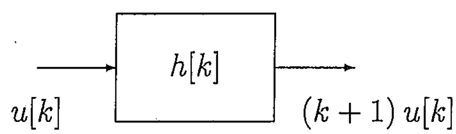
Give a formula for the impulse response $h(t)$.



$h(t) = \delta(t) - \delta(t - t_0)$

step response of a tapped delay line with two taps $\Rightarrow$ $p(t) = u(t) - u(t - t_0)$

(b) Discrete-Time. 10 points.
For the block diagram below,



Give a formula for the impulse response $h[k]$.

$h[k] = u[k]$ since $u[k] * u[k] = (k+1)\,u[k]$

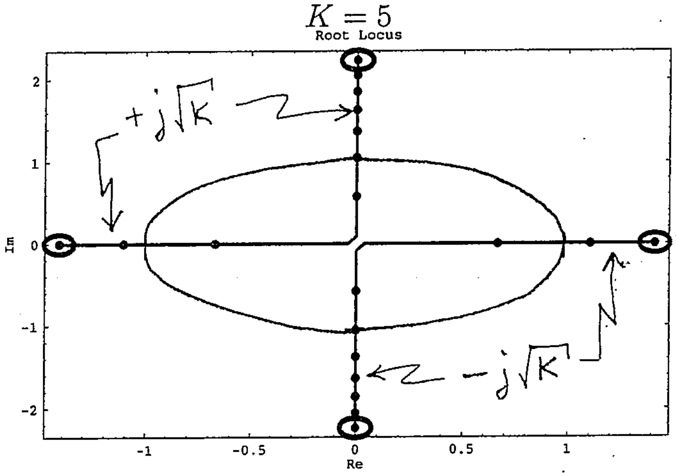
Problem 1.5 Discrete-Time Stability. 15 points.

A linear time-invariant *discrete-time* system with input $f[k]$ and output $y[k]$ is described by the following difference equation

$$y[k] \; + \; K\, y[k-2] = f[k]$$

where K is a real-valued parameter. Below is the plot of the roots for $K \in [-2, 5]$. The horizontal axis is the real part of the root, and the vertical axis is the imaginary part of the root.

$K = 5$

$K = -2$  $K = -2$

$K = 5$

Due to the aspect ratio of the plot, the unit circle looks like an ellipse.

(a) What are the characteristic roots? 5 points. Double root when $K=0$

$$1 + K\gamma^{-2} = 0$$

$$K\gamma^{-2} = -1$$

$$\gamma^2 = -K \Rightarrow \gamma = \pm j\sqrt{K}$$

(b) What range of K makes the system stable? 10 points.

The roots need to lie inside the unit circle for stability.

With $\gamma_0 = +j\sqrt{K}$, $|\gamma_0| < 1 \Rightarrow K > -1$ for $K \le 0$

Sames goes for $\gamma_1 = -j\sqrt{K}$ $K < 1$ for $K \ge 0$

$$-1 < K < 1$$

K-17